Engineering Notes

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Jet Interaction Wrap-Around on Bodies of Revolution

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Nomenclature

A = area
d = diameter
G = function defined in text
h = penetration or separation height (Fig. 1)
J = parameter from blast wave analogy
K = amplification factor
M = Mach number
P = pressure
r = shock trace radius (Fig. 1)
R = separation line radius (Fig. 1)
x = distance from point of injection (Fig. 1)
γ = ratio of specific heats
σ = separated region equivalent wedge angle
ϕ = angular location of separation line (Fig. 3)

Subscripts

b, fp = body and flat plate, respectively
j, o = jet and stagnation conditions, respectively
s, t = separation line and shock trace, respectively
$2, \infty$ = separation region and freestream, respectively
$*$ = sonic conditions

Introduction

When a round jet exhausts from a body of revolution, the resulting interaction flowfield is curved about both the jet and body axes. This Note presents a method for estimating the extent and shape of the three-dimensional flowfield for a sonic jet exhausting normal to a supersonic freestream. Existing experimental data for confined jets issuing from flat plates, and the blast-wave analogy, are used to develop expressions for the interaction curvature about the jet axis. To account for the curvature about the body (wrap-around), a direct analogy is drawn between straight lines on the flat plate and circular arcs on the body of revolution. That is, the flat plate interaction is simply wrapped around the body. This method of accounting for body curvature, as yet only marginally verified by experimental evidence, is common in the literature for jet interactions on bodies of revolution.[1-4] Considerable loss in the gross side force results from wrap-around. The unfavorable distribution of pressures within the interaction region can be such that the total side force is less than that which would be expected from the thrust of the jet alone. In this paper, the shape and extent of the wrap-around of the interaction region is estimated and a comparison is made between the existing side force data for round jets issuing from flat plates and from bodies of revolution. An estimate is provided for the minimum body diameter below which the effects of wrap-around on side force should be considered.

Received August 18, 1969; revision received December 5, 1969. This work performed for the Propulsion Development Department, Naval Weapons Center, China Lake, Calif.

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Flat-Plate Flowfield and Separation Model

For the flat plate flowfield (Fig. 1), the shape of the trace (intersection with the plate) of the freestream bow shock in the absence of the freestream boundary layer can be obtained from the blast-wave analogy. The form which is most useful is[5]

$$r(x)/d_j = G(\gamma_\infty, M_\infty)(P_{oj}/P_\infty)^{1/4}[(x_t/d_j)]^{1/2} \tag{1}$$

where

$$G(\gamma_\infty, M_\infty) = [(\gamma_\infty/J)(P_\infty/P_{o\infty})(A_\infty/A^*_\infty)]^{1/4}$$

In this expression $\gamma_\infty/J = 1.4/0.88$ and J is a parameter from the blast-wave analogy. Values for $G(\gamma_\infty, M_\infty)$ are shown in Fig. 2. The presence of the separated boundary layer is taken into account by assuming that the shape of the line of separation, $R(x)/d_j$, is similar to that of the freestream bow shock trace and is calculated from Eq. (1) by substituting x_s for x_t. The distance x_s from the jet origin to the point of separation is determined by semiempirical methods. For a turbulent boundary layer the normalized pressure in the separated region may be estimated, according to Zukoski,[6] as $P_2/P_\infty = 1 + (M_\infty/2)$. Using the tangent wedge relations, as suggested by Barnes et al.,[7] the wedge angle σ necessary to induce this pressure rise may be computed. From geometry,

$$-(x_s/d_j) = (h_s/d_j) \cot\sigma \tag{2}$$

Finally, an approximate form of the formula suggested by

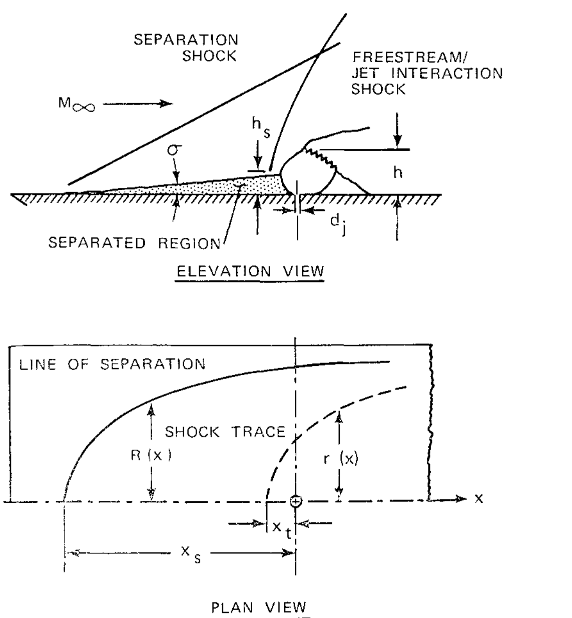

Fig. 1 Sketch of flat-plate interaction.

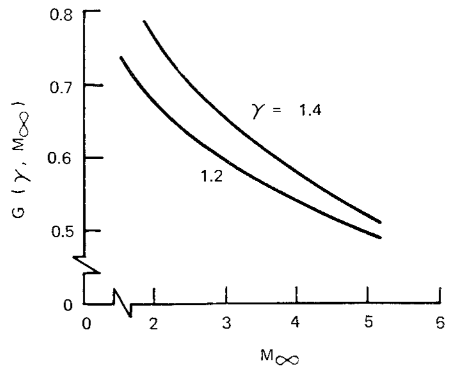

Fig. 2 The function $G(\gamma_\infty, M_\infty)$.

Zukoski and Spaid[8] gives

$$h_s/d_j \approx (h_s/h)(P_{oj}/P_\infty)^{1/2}/M_\infty \quad (3)$$

When (h_s/h) is known, the foregoing relationships provide sufficient information for predicting the shape of the flat plate interaction. The wrap-around effects thus calculated will be conservative (large) since Eq. (3) gives a somewhat larger effective jet penetration in comparison with other expressions.[9,10] The method outlined here can be used to calculate the interaction field for laminar freestream boundary layers with the expense of some complication due to a dependence on the freestream Reynolds number.

Wrap-Around

It is assumed now that the flat plate flowfield can be simply wrapped around the body. Among other things, this assumes that the disturbance along the generatrix of the cylinder which passes through the jet origin is not overly dependent upon the curving of the flow. To accomplish the transformation, set $\phi = 2R/d_b$. The result is

$$\phi = 2G(\gamma_\infty, M_\infty)(P_{oj}/P_\infty)^{1/4}[(x/d_j) - (x_s/d_j)]^{1/2}d_j/d_b \quad (4)$$

where ϕ now defines the angular location of the separation line as shown in Fig. 3.

Figure 4 illustrates the relative effects of P_{oj}/P_∞ and M_∞ upon the extent of wrap-around. Extensive wrap-around can occur upstream of the point of injection; therefore, locating the jet at the base of the body does not eliminate the problem. Although the model used here is less valid at $M_\infty < 3$, the trend of increasing severity of wrap-around as M_∞ is reduced is significant. Another qualitative conclusion is that forward located jets may produce disturbances which significantly affect the forebody flow, even to the point of altering the structure of the nose shock.

A number of other interesting trends can be indicated through the use of Eq. (4). For instance, when $x = 0$ Eqs. (2–4) indicate that $\phi \propto (P_{oj}/P_\infty)^{1/2}$. The exponent decreases slightly for larger values of x. The method may also be used in order to estimate the proper positioning of fins or strakes for the purpose of controlling wrap-around. In this connection, it is important to note that the fin span is also a significant factor[11] since the separated region can spill over the tips as well as the leading edges of such devices.

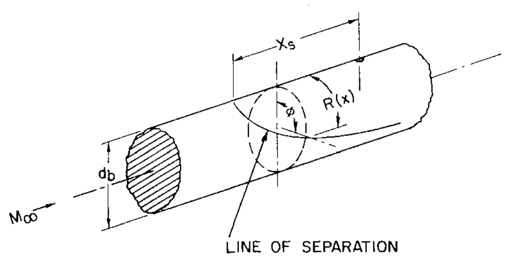

Fig. 3 Coordinate transformation for wrap-around.

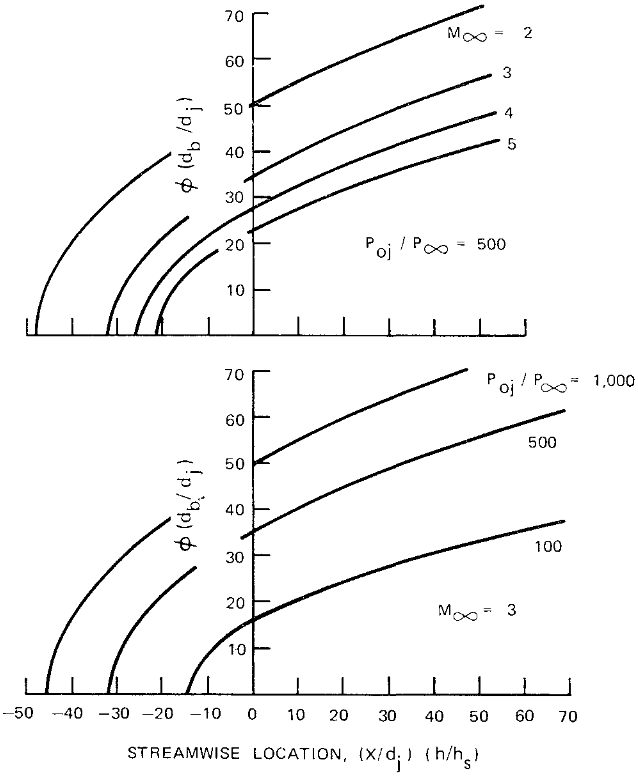

Fig. 4 Wrap-around with P_{oj}/P_∞ and M_∞ as parameters.

Correlations of Existing Force Data

In a recently completed study, the writer and R. D. Ulrich† have accumulated the experimental results of a number of investigators (Table 1). The results suggest collectively that the amplification factor associated with a flat plate in a laminar flow is

$$K_{fp} = 1.54M_\infty^{0.25}(P_{oj}/P_\infty)^{-0.036}G(\gamma_\infty, M_\infty)^{0.36} \quad (5)$$

and for a body of revolution

$$K_b = 0.75M_\infty^{0.39}(P_{oj}/P_\infty)^{-0.25}G(\gamma_\infty, M_\infty)^{-0.23}(d_b/d_j)^{0.46} \quad (6)$$

These expressions are for normal sonic jets exhausting from a point near the downstream edge (tail) of a body at zero angle of attack. (The results obtained should be qualitatively applicable to jets at forward locations on finless bodies at zero angle of attack.) The flat plate expression for a laminar freestream boundary layer is used since this condition results in the largest upstream interaction region and (it is thought)

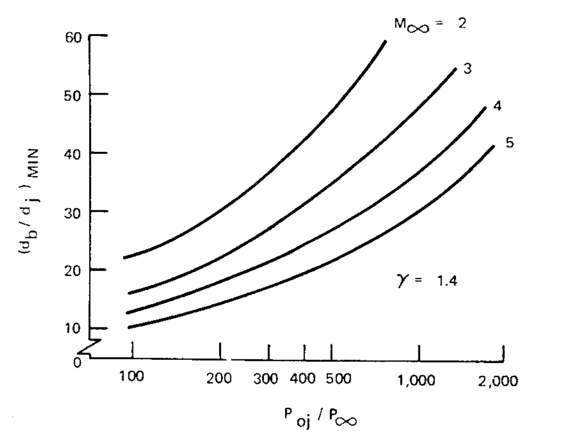

Fig. 5 Body diameter necessary for wrap-around to be negligible.

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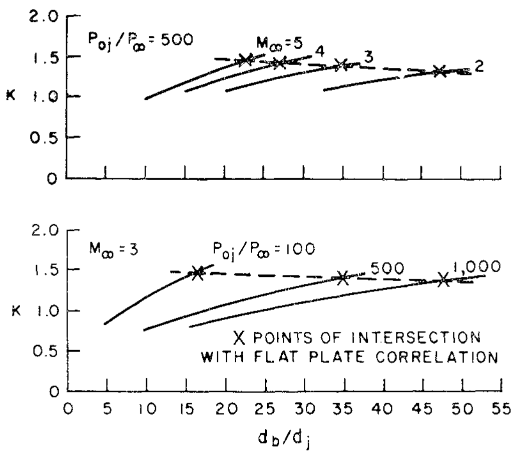

Fig. 6 Amplification factors with P_{oj}/P_∞ and M_∞ as parameters.

more severe wrap-around effects than for transitional or turbulent flows.

Table 1 shows the references and ranges of parameters for the data from which Eqs. (5) and (6) were derived. A discussion of the rationale used in these correlations is beyond the scope of this Note except to say that the "effective penetration" of the jet was a major criterion and that the blast-wave analogy was used to correlate the effects of the curvature of the interaction region around the jet axis (not to be confused with wrap-around). The correlations are tentative and should not be used without due consideration to their ranges of applicability and the extent to which they are verified by the data.

Estimation of Minimum Body Diameters

In spite of the tentative nature of the correlations, they can be useful in providing some first estimates of the effect of wrap-around on the performance of body of revolution jet steering (JS) systems in terms of the side force produced. It is assumed that at some large value of d_b/d_j the system will yield side forces that are not significantly less than those obtained with a flat plate. To estimate this point of equivalency, K_{fp} is equated with K_b and Eqs. (5) and (6) are solved for d_b/d_j which, it is hypothesized, represents a value below which significant losses can be expected due to wrap-around. The result is

$$(d_b/d_j)_{\min} = 4.78M_\infty^{-0.30}(P_{oj}/P_\infty)^{0.46}G(\gamma_\infty,M_\infty)^{1.28} \quad (7)$$

Figure 5 illustrates Eq. (7) for $\gamma = 1.4$ and a representative range of P_{oj}/P_∞ and M_∞. Figure 6 shows plots of Eq. (6) that illustrate the nature of the intersections of this expression with the horizontal lines obtained from Eq. (5) and the separate effects of P_{oj}/P_∞ and M_∞ upon the location of the intersection. Inasmuch as Eq. (6) for the body-of-revolution does not asymptotically approach to horizontal lines in Fig. 6 at high values of d_b/d_j, its use in the preceding hypothesis is inaccurate. The errors involved, however, are not large, especially when considered in the light of other uncertainties.

Example and Concluding Remarks

To illustrate the use of these tools, consider, for example, a sonic jet with P_{oj} = 1000 psia issuing from a body of revolution at 30,000 ft and $M_\infty = 4$. For a thrust from the jet alone of approximately 300 lb, a nozzle of approximately $\frac{1}{2}$-in. diam would be required. Under these conditions $P_{oj}/P_\infty \approx 230$, so that Fig. 5 gives $d_b/d_j \approx 19$. The conclusion is that if the vehicle diameter (d_b) exceeds 9.5 in., there will be only slight benefit in attempting to correct for wrap-around effects. Examples such as this can, of course, be generalized to apply to broader areas of application.

Unfortunately, it is not presently possible to obtain quantitative predictions of JS system performance in the "gray" areas between the flat plate and the body of revolution. In the absence of such information, methods such as the one presented here may be useful as aids to the designer of JS systems on more complicated bodies.

Table 1 Data sources

Ref.	M_∞	P_{oj}/P_∞	d_j, in.	d_b/d_j
12	3.48	15–200	0.25	Flat plate (laminar)
	3.71	40–540	0.25	
	3.85	40–1240	0.13	
		25–300	0.25	
	4.51	30–750	0.25	
12	3.99	70–915	0.125	12.0
		30–370	0.281	5.3
11	3.97	55–850	0.119	16.8
		45–730	0.160	12.5
		60–460	0.224	8.9
	3.98	180–870	0.25	14.0
2	3.98	180–870	0.25	14.0
13	2.84	13–176	0.159	12.6
		36–181	0.221	9.0
	3.90	180–820	0.159	12.6

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